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Extension of the Incompressible $n = 2$ Vortex into Compressible

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Nomenclature

c_p, c_v = heat capacities constant pressure and volume, respectively, $W \cdot s/kg \cdot K$
H = normalized axial velocity component, βReh
k = thermal conductivity, $W/m \cdot K$
L = large dimensionless number (10,000)
M_o = vortex Mach number, $V_{\theta c}/\sqrt{\gamma R T_\infty}$
n = vortex exponent of the n-family
O = order of magnitude
p = static pressure, Pa
Pr = Prandtl number, $\mu c_p/k$
$\boldsymbol{q}$ = total velocity vector, m/s
R = gas constant, $J/kg \cdot K$
Re = vortex Reynolds number, $\rho_\infty V_{\theta c} r_c/\mu$
r, θ, z = radial, azimuthal, and axial coordinates, m
r_c = core radius, m
s = entropy, $W \cdot s/kg \cdot K$
T = static temperature, K
t = dimensionless dummy variable
U = normalized radial velocity component, βReu
u = dimensionless radial velocity component, $V_r/V_{\theta c}$
V = normalized tangential velocity component
V_r, V_θ, V_z = radial, tangential and axial velocity components, m/s
$V_{\theta c}$ = core tangential velocity, $\Gamma_\infty/2\pi r_c$, m/s
w = dimensionless axial velocity component, $V_r/V_{\theta c}$, zh
β = dimensionless density, ρ/ρ_∞
Γ_∞ = vortex circulation, m^2/s
γ = specific heat ratio, c_p/c_v
Δs = dimensionless entropy change, $(s - s_\infty)/c_p$
Θ = dimensionless static temperature, T/T_∞
ϑ = dimensionless dummy variable
κ = vortex strength, $\Gamma_\infty/2\pi$, m^2/s
μ = fluid viscosity, $kg/m \cdot s$
ξ, ζ = dimensionless coordinates, r/r_c and z/r_c
Π = dimensionless static pressure, $p/\rho_\infty V_{\theta c}^2$
ρ = fluid density, kg/m^3

Subscripts

c = vortex core
∞ = properties far away from the vortex center

I. Introduction

THE seeds in compressible vortex research appeared in the early thirties with Taylor's theoretical paper on the isentropic potential vortex [1]. Work on the same subject reemerged again in the midfifties in connection with studies on the vortex/shockwave interaction [2,3] and has continued ever since. Contributions to the confined type were mainly the result of NASA's gaseous nuclear rocket motor project [4] and the refrigeration effect due to temperature drop in a Ranque–Hilsh tube [5]. In another front Meager [6] explored approximate solutions of isentropic swirling flow through a nozzle. Rott [7] discussed the temperature profile in a steady Burgers vortex under rather restrictive conditions. Mack [8] examined analytically the compressible, viscous, heat-conducting vortex, created by a rotating cylinder in a domain of infinite extend. Later, Bellamy-Knights [9] extended Mack's solution to account for the radial flow. Sibulkin [10] analyzed the decay of Taylor's vortex for low Mach number conditions. Brown [11] advanced Hall's [12] theory on three-dimensional vortices including density variations. Colonius et al. [13] examined the compressibility effects of decaying, unconfined vortices, taking into consideration the contributions of viscosity and heat transfer. Bagai and Leishman [14] visualized via the density gradient technique helicopter blade vortex structures stipulating isentropic flow conditions. Chiocchia [15] and Ardalan et al. [16] obtained solutions to ideal compressible vortices using hodograph plane transformations. Von Ellenrieder and Cantwell [17] studied the self-similarity of slightly compressible free vortices. Orangi et al. [18] reported on the numerical solution of a family of viscous, compressible, heat-conducting, self-similar-slender vortices. Perez-Saborid et al. [19] analyzed the evolution of unconfined vortices including the thermal aspects; Melville [20] examined numerically the compressibility effects on the breakdown of free vortices, whereas Rusak and Lee [21,22] looked at their stability in pipes. Observations on vortices of this kind are due to Mandella [23], Kalhoran and Smart [24], and Cattafesta and Settles [25].

In this brief article we deal with the extension of the incompressible $n = 2$ vortex [26,27] into compressible. As usual, density variations are taken into account through the inclusion of the energy equation and the ideal gas law. Based on the intense vortex constraint [26,27] the governing equations are simplified, rendering part of the hydrodynamic problem analogous to the incompressible version and as such the velocity is determined through a straightforward variable transformation. The temperature is deduced from the energy equation whereas the density and pressure are calculated from the radial momentum and state equations. Actual compressible vortices possess complex properties. Nevertheless, simplified models like the present can capture some of their most salient physical characteristics. Consequently, fundamental contributions in this area are relevant to a variety of technologically important applications such as, for example, in aeroacoustic research, vortex stability, nonintrusive compressible vortex characterization via shadowgraphy, and

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others. The primary limitations of the present approach are the same to those associated with the classical incompressible vortex representation of Burgers [28] and Burgers-like vortex formulations such as these of Rott [29], Sullivan [30], and Bellamy-Knights [31]. For example, the tangential and radial velocity components are only functions of the radial coordinate, whereas the axial velocity varies linearly with z. At $z = 0$ plane, the last velocity is zero whereas the other two are allowed to freely slip. In addition, the radial component is either linear or asymptotically linear, thus making the latter velocity to grow without bounds as $r \to \infty$. In the present formulation, however, because the radial velocity component is bounded, it requires no drastic assumptions to obtain finite solutions for the temperature [7].

II. Formulation of the Problem

Consider the motion of a steady, compressible, axisymmetric vortex. We are interested in solutions of a particular simple class of vortices [26,27] in which the velocity is of the general form $\boldsymbol{q}[V_r(r), V_\theta(r), V_z = z\,\mathrm{fn}(r)]$. Starting points are the simplified equations of continuity, Navier–Stokes, energy, and the equation of state of a calorically perfect gas. Because we are dealing with strong vortices, the traditional assumption requiring that u and $h \ll V$ is implemented [26,27]. Under these conditions the governing equations are

Conservation of mass:

$$\frac{1}{\xi}\frac{\partial \beta u \xi}{\partial \xi} + \beta h = 0 \tag{1}$$

Radial momentum:

$$\frac{\beta V^2}{\xi} = \frac{\partial \Pi}{\partial \xi} \tag{2}$$

Tangential momentum:

$$Re\frac{\beta u}{\xi}\frac{\partial V\xi}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{1}{\xi}\frac{\partial V\xi}{\partial \xi}\right) \tag{3}$$

Axial momentum:

$$\frac{\partial \Pi}{\partial \zeta} \approx 0 \tag{4}$$

Energy equation:

$$\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\right) + Pr(\gamma-1)M_o^2 f = \beta Pr Re u\frac{\mathrm{d}\Theta}{\mathrm{d}\xi} - Pr Re(\gamma-1)M_o^2 u\frac{\mathrm{d}\Pi}{\mathrm{d}\xi} \tag{5}$$

Where

$$f = \xi^2\left[\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{V}{\xi}\right)\right]$$

Equation of state:

$$\Pi = \frac{\beta\Theta}{\gamma M_o^2} \tag{6}$$

The simplification of the governing equations has been achieved based on the same order of magnitude considerations as were outlined in [26,27].

From Eq. (4) we deduct that the pressure should be a sole function of ξ. The density from the ξ-momentum and the temperature from the equation of state must also depend on ξ alone.

Letting $U = \beta Re u$ and $H = \beta Re h$, and replacing the pressure gradient by its equivalent $\beta V^2/\xi$ in the energy [Eq. (5)], the preceding set of Eqs. (1–5) transforms into

Continuity:

$$\frac{1}{\xi}\frac{\mathrm{d}U\xi}{\mathrm{d}\xi} + H = 0 \tag{7}$$

Radial momentum:

$$\frac{\beta V^2}{\xi} = \frac{\mathrm{d}\Pi}{\mathrm{d}\xi} \tag{8}$$

Tangential momentum:

$$\frac{U}{\xi}\frac{\mathrm{d}V\xi}{\mathrm{d}\xi} = \frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{1}{\xi}\frac{\mathrm{d}V\xi}{\mathrm{d}\xi}\right) \tag{9}$$

Energy:

$$\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\right) - PrU\frac{\mathrm{d}\Theta}{\mathrm{d}\xi} = -Pr(\gamma-1)M_o^2\left(f + \frac{UV^2}{\xi}\right) \tag{10}$$

The *equation of state* stays as is.

The required boundary conditions are

1) $\xi = 0$, $V(\xi) = U(\xi) = 0$, and $\mathrm{d}H(\xi)/\mathrm{d}\xi = \mathrm{d}\Theta(\xi)/\mathrm{d}\xi = 0$

2) $\xi \to \infty$, $V(\xi)\xi$, $\Theta(\xi)$, $\gamma M_o \Pi(\xi)$, and $\beta(\xi) \to 1$

The hydrodynamic portion that consists from Eqs. (7) and (9) has now exactly the same form as its incompressible counterpart [26,27]. By analogy, the most practical member of the many known solutions is

$$U = -\frac{6\xi^3}{1+\xi^4}, \qquad H = \frac{24\xi^2}{(1+\xi^4)^2}, \quad \text{and} \quad V = \frac{\xi}{(1+\xi^4)^{1/2}} \tag{11}$$

All the preceding velocity components reduce asymptotically to zero as $\xi \to \infty$. Because the actual radial and axial components involve the density β, both will be affected by compressibility. On the other hand the tangential will not be influenced by β.

Integrating twice the energy Eq. (10) and applying the temperature boundary conditions at $\xi = 0$ and $\xi \to \infty$, assuming $Pr = 2/3$, gives

$$\Theta(\xi) = 1 + \frac{(\gamma-1)M_o^2}{6}\left(\arctan(\xi^2) - \frac{\xi^2}{1+\xi^4} - \frac{\pi}{2}\right) \tag{12}$$

It is easily verifiable that the expression given by Eq. (12) is an the exact solution of the energy Eq. (10).

The value of Prandtl number for most of the common gases in 1 atm and 300 K is about 0.7. Because the latter dimensionless number does not influence strongly the solution, we will use the expression given by Eq. (12) to represent Θ (around $Pr = 0.7$), without suffering any substantial physical loss.

Differentiating Eq. (6) with ξ we have

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\xi} = \frac{1}{\gamma M_o^2}\left(\Theta\frac{\mathrm{d}\beta}{\mathrm{d}\xi} + \beta\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\right) \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}\xi}\ln(\beta\Theta) = \gamma M_o^2\frac{V^2}{\xi\Theta} \tag{13}$$

Integration of the second member of the preceding set of equations and application of the boundary conditions for the density yields

$$\beta = \frac{\exp\left[\gamma M_o^2\left(\int_0^\xi \frac{V^2}{\xi\Theta}\,\mathrm{d}\xi - \lim_{L\to\infty}\int_0^L \frac{V^2}{\xi\Theta}\,\mathrm{d}\xi\right)\right]}{\Theta} \tag{14}$$

The pressure is then determined from the first expression of Eq. (13) along with Eqs. (12) and (14). The preceding integrals are easily evaluated to a good degree of accuracy by any of the standard numerical methods like Romberg's, or better the application of software like Mathematica, MAPLE, or MATLAB.

III. Results and Discussion

The analysis assumes a perfect gas, which implies that all the material properties are constant. In reality, however, properties such as c_p, k, and μ do depend on the temperature. To remain pragmatic, the applicability of the solution should be limited to small temperature variations, say around 300 K.

Multiplying Eq. (10) by ξ and then integrating once, realizing that the temperature derivative at $\xi = 0$ must be zero, we obtain

$$\underbrace{-\xi\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}}_{g_1} + \underbrace{Pr\int_0^\xi U\xi\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\,\mathrm{d}\xi}_{g_2} - \underbrace{Pr(\gamma-1)M_o^2\int_0^\xi UV^2\,\mathrm{d}\xi}_{g_3} \underbrace{+\,Pr(\gamma-1)M_o^2\int_0^\xi \xi f\,\mathrm{d}\xi}_{g_4} = 0 \qquad (15)$$

where g_1, g_2, g_3, and g_4 represent the heat conduction, convection, cooling of fluid element due to expansion, and the heat generated by viscous action, respectively.

The magnitude of the different terms in the Eq. (15) are shown in Fig. 1d. There are four processes in action. In order for the flow to balance the centrifugal force, the pressure must increase with the radius. Material elements carried by the converging field (g_3) in the direction of decreasing pressure expand and therefore their temperature ought to drop, attaining a minimum at the center where the temperature must obey the symmetry boundary condition. The effects of heat conduction (g_1) and heat generation (g_4) due to friction are not sufficient to compensate for the temperature dive. Finally, the converging flow carries with it energy (g_2) required to balance the three other effects and thus keeping the temperature steady.

Solutions for ideal conditions such as the isentropic flow are useful because they provide a point of origin, define a sense of direction, and often furnish acceptable approximations to reality. Under the last assumption, the processes in all fluid elements that make up the flowfield must be reversible and adiabatic, i.e., g_1, g_2, g_3, and g_4 must be zero. The first will be met if the fluid is nonconducting, i.e., when $k = 0$. The second and the third will also be satisfied if there is no fluid convection in the ξ-direction ($u = 0$), whereas the fourth will be attained if the flow is frictionless ($\mu = 0$). Then, from continuity $h = 0$, ζ-momentum and energy will be automatically satisfied. The ξ-momentum is still given by Eq. (8) whereas the θ-momentum is satisfied regardless of the tangential velocity shape. The required additional equation is the thermodynamic relation for isentropic flow, $\Pi = \beta^\gamma/(\gamma M_0^2)$. Using the last expression along with the ξ-momentum and state equations, for an $n = 2$ vortex, the exact solutions for the pressure, temperature, and density are

$$\Pi = \frac{1}{\gamma M_o^2}\left[1 - \frac{\gamma-1}{2}M_o^2\left(\frac{\pi}{2} - \operatorname{arc\,tg}\xi^2\right)\right]^{\gamma/(\gamma-1)}$$
$$\Theta = 1 - \frac{\gamma-1}{2}M_o^2\left(\frac{\pi}{2} - \operatorname{arc\,tg}\xi^2\right) \qquad (16)$$
$$\beta = \left[1 - \frac{\gamma-1}{2}M_o^2\left(\frac{\pi}{2} - \operatorname{arc\,tg}\xi^2\right)\right]^{1/(\gamma-1)}$$

The density expression is similar to that derived by Bagai and Leishman [14]. Note that the isentropic assumption, as anticipated, does not involve the Prandtl number.

The present theoretical deduction suggesting that the tangential velocity must retain the shape of an incompressible vortex is reasonably supported by past experiments [23–25]. Consequently, properties such as vorticity and circulation remain the same as those of the incompressible case [31]. Because the mechanical dissipation does not involve the density, the total frictional losses per unit height will be also equal to the incompressible $n = 2$ vortex [32]. The pressure, density, and temperature profiles for a range of Mach numbers are given in Figs. 1a–1c. It is evident that all three properties decrease along the flow direction attaining minimum values at the

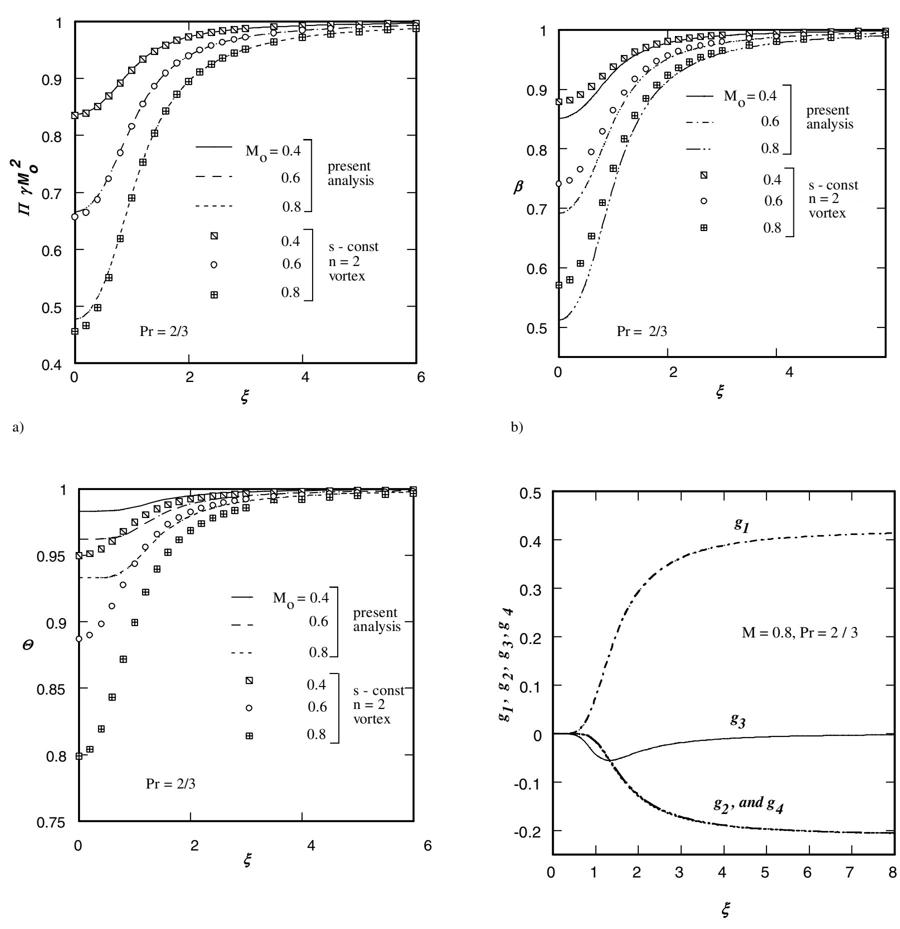

Fig. 1 a) Static pressure, b) density, c) temperature, and d) terms in the energy Eq. (15).

vortex center. Under the isentropic flow constraint, all curves have similar shape as the more general case. Besides the static pressure, the homentropic hypothesis overestimates the temperature and underestimates the density drop.

The meridional flow velocity components keep the shapes of the incompressible vortex but increase in magnitude. As the fluid approaches the center, fluid elements dilate and thus the density drops. To conserve mass the fluid compensates for by a velocity increase. The larger the vortex Mach number is, the steeper the density reduction, leading into a further intensification of these velocity components.

In addition to the previously treated conservation laws, every mathematical representation of reality must respect the *second law of thermodynamics*:

$$\Theta PrU\frac{\mathrm{d}\Delta s}{\mathrm{d}\xi}=\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\right)+Pr(\gamma-1)M_o^2 f$$

or

$$\Delta s=-\frac{1}{3}(\gamma-1)M_o^2\left(\int_0^{\xi}\frac{(\vartheta^4+3)\vartheta}{\Theta(1+\vartheta^4)^2}+\lim_{L\to\infty}\int_0^{L}\frac{(\vartheta^4+3)\vartheta}{\Theta(1+\vartheta^4)^2}\,\mathrm{d}\vartheta\right)$$

It is not difficult to see that the entropy in this particular case rises along the flow direction (negative ξ-direction), thus confirming that the present mathematical model obeys also the *second law*.

IV. Conclusions

In this paper we have extended the incompressible $n=2$ vortex model to account for density variation. Because all the velocity components are bounded the temperature profile is obtained without the imposition of limiting assumptions. The basic elements of the approach could now be employed to examine phenomena such as vortex stability, compressible flow aeroacoustics, compressible vortex visualizations via shadowgraphy, and several others.

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